

Phase-stable limited relativistic acceleration or unlimited relativistic acceleration in the laser-thin-foil interactions

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Abstract

To clarify the relationship between phase-stable acceleration (PSA) and unlimited relativistic acceleration (URA) (Phys. Rev. Lett. 104, 135003 (2010)), an analytical relativistic model is proposed in the interactions of the ultra-intense laser and nanometer foils, based on hydrodynamic equations. The dependence of the ion momentum on time is consistent with the previous results and checked by PIC simulations. Depending on the initial ion momentum, relativistic RPA contains two acceleration processes: phase-stable limited relativistic acceleration (PS-LRA) and URA. In PS-LRA, the potential is a deep well trapping the ions. The ion front, i.e., the bottom, separates it into two parts: the left half region is PSA region; the right half region is PSD region, where the ions climb up and are decelerated to return back. In PS-LRA, the maximum ion energy is limited. If the initial ion momentum large enough, ions will experience a potential downhill and drop into a bottomless abyss, which is called phase-lock-like position. URA is not phase-stable any more. At the phase-lock-like position the ions can obtain unlimited energy gain and the ion density is non-zero. You cannot have both PSA and URA.

Laser-ion acceleration has been an international research focus[1–4], however it is still a challenge to obtain mono-energetic proton beams larger than 100MeV. As a promising method to generate relativistic mono-energetic protons, radiation pressure acceleration (RPA) has attracted more attention[3–6] and becomes dominant in the interaction of the ultra-intense laser pulse with nanometer foils. The phase-stable acceleration[7] predicted that the energy spread can be improved in the interactions of nanometer-foils with circular-polarized laser pulses. With thin-shell model[8], Bulanov and coworkers[8] pointed out that the ions can obtain unlimited energy gain by RPA in the relativistic limit. Yan and coworkers tried to predict the ion energy distribution with a self-similar hydrodynamic theory[9]. However, it is nonrelativistic and under the plasma approximation which allows $\nabla \bullet E \neq 0$ and $n_i - n_e = 0$ satisfy together, where E is the acceleration field and n_i (or n_e) is the ion (or electron) density. However, can the ions obtain unlimited energy gain in the phase-stable region or is the phase-stable acceleration still possible in the relativistic limit? How about the relationship between them or what are the critical conditions for them? To give clear answers of the questions, an analytic self-consistent relativistic fluid model is proposed to describe the relativistic radiation pressure acceleration and to recheck the unlimited ion-acceleration region.

The ion acceleration in the interaction of ultra-intense laser and nanometer foils contains two stages: the hole-boring process and the radiation pressure acceleration. Here the transition of them is assumed steady and the instability of the acceleration sheath is suppressed well, which can be realized for specially designed targets [10, 11]. In the hole-boring process, the ion velocity can reach u_{hb} [12, 13], which is the hole-boring velocity and also the initial velocity of the ions in the radiation pressure acceleration. The initial time, t_0 is when the compression layer is detached from the foil. It is decided by the target thickness and u_{hb} . With the initial conditions: u_{hb} and t_0 , the dependence of the ion energy on time can be obtained and consists with the results of thin-shell model[3] and has been checked by PIC simulations[3]. Depending on the laser intensity, the initial ion momentum will determine two different acceleration processes: the phase-stable limited relativistic acceleration (PS-LRA) and the unlimited relativistic acceleration (URA). When the initial ion momentum is smaller than the critical one, the potential is a deep well trapping the ions. The acceleration mode is PS-LRA and contains the phase-stable acceleration region (PSA) and the phase-stable deceleration region (PSD). The well is separated into two half-regions: the left

half-region and the right half-region by the bottom, which is the ion front. The left half-region is PSA region, where the electric field is positive and ions coast down to the bottom. While the right half-region is PSD region, where the electric field is negative and ions climb up the potential uphill and are decelerated to return to the bottom. The deceleration is also phase-stable. No matter in PSA or PSD region, the maximum ion energy is limited and ascertained by an analytical formulation. Since PSA and PSD are separated by the ion front where the ion density is zero, the ions in two regions can not exchange from each other. If the initial ion momentum is large enough, the ions can get across the potential uphill and experience a potential downhill and then drop into the bottomless abyss which is the phase-lock-like position. If the ions can reach the phase-lock-like position, they can obtain unlimited energy gain as Bulanov and coworkers pointed out[8]. The acceleration mode is URA and not phase-stable any more. The electron density is smaller than the ion density and the electron front increases with time. The phase-lock-like position of URA is the limiting ion front and the ion density is non-zero as time tends to zero. You cannot have both PSA and URA in the same acceleration process. Therefore, the maximum ion energy is finite in the phase-stable region and URA is not phase-stable any more. However, no matter in PS-LRA or URA, the plasma tends to neutral as time tends to infinite.

For convenience, the physical parameters: the time, t , the ion position, x , the ion velocity, v , the electron field, E , the electric potential, φ , the plasma density, n , and the light speed, c , are normalized as follows: $\tau = \omega t, \hat{x} = xk, u = v/c, \hat{E} = E/E_0, \phi = \varphi/\varphi_0, \hat{n} = n/n_0$, where n represents n_i (or n_e), n_0 is the reference density, ω is the light frequency, $k = \omega/c$ is the wave number, c is the light speed, $E_0 = k\varphi_0$, $e\varphi_0 = \gamma_{em}m_e c^2$ and γ_{em} is the maximum electron energy. Here e is the elemental charge.

With reference to the results given by Mako and Tajima in Ref. [1], in the self-similar state, the density distribution of ions is assumed as:

$$\hat{n}_k = \frac{1}{\Sigma Q_k} (1 + \phi)^\alpha, k = 1, \dots, N, \quad (1)$$

where the subscribe $_k$ stands for the ion species, Q_k is the charge number of the k th species ion, the index α depends on the laser intensity and the target thickness and discussed in the Ref. [9].

With the transformation: $\xi = \hat{x}/\tau$, the normalized continuity and motion equation of

ions and Poisson's equation are given as:

$$\begin{aligned} (u_k - \xi) \frac{\partial \ln \hat{n}_k}{\partial \xi} &= -\frac{\partial u_k}{\partial \xi}, \\ (u_k - \xi) \frac{\partial \gamma_k u_k}{\partial \xi} &= -\beta_k \frac{\partial \phi}{\partial \xi}, \end{aligned} \quad (2)$$

$$\frac{1}{\tau^2} \frac{\partial^2 \phi}{\partial \xi^2} = -\rho (\Sigma Q_k \hat{n}_k - \hat{n}_e) \quad (3)$$

where $\beta_k = \frac{Q_k \gamma_{em} m_e}{M_k}$, M_k is the mass of certain ions, $\gamma_k = (1 - u_k^2)^{-1/2}$, $\rho = \frac{\omega_{pe}^2}{\gamma_{em} \omega^2}$, and $\omega_{pe}^2 = \frac{n_0 e^2}{\epsilon_0 m_e}$.

Solving Eq. (2), the ion velocity satisfies:

$$\xi + \left(\frac{\gamma_k}{\gamma_{k,0}} \right)^{-3/2} u_{k,0} = u_k + \frac{\gamma_k^{-3/2}}{2\alpha} (\chi - \chi_0), \quad (4)$$

and the potential in the ion region is given by:

$$\phi_1 = \frac{\left(\chi - \chi_0 - 2\alpha u_{k,0} \gamma_{k,0}^{3/2} \right)^2}{4\alpha \beta_k} - 1, \quad (5)$$

where $\gamma_{k,0} = (1 - u_{k,0}^2)^{-1/2}$, $\chi_0 = \chi(u_{k,0})$,

$$\chi = \int_0^{u_k} \gamma_k^{3/2} du_k, \quad (6)$$

and $u_{k,0}$ is the hole-boring velocity given by[12, 13]:

$$u_{k,0} = \frac{u_{hb}}{c} = \sqrt{\frac{Z}{A} \frac{m_e}{\gamma_{hb} M_k} \frac{n_c}{2n_0} a} \quad (7)$$

where $a^2 = 0.732 I_{10^{18} \text{W/cm}^2} \lambda_{\mu\text{m}}^2$, $I_{10^{18} \text{W/cm}^2}$ is the laser intensity in unit of 10^{18}W/cm^2 , and $\gamma_{hb} = (1 - (u_{hb}/c)^2)^{-1/2}$. The beginning time τ_0 is when the compressed ion and electron layer is detached from the foil and given by[13]:

$$\tau_0 = \frac{d}{\lambda} \frac{2\pi}{u_{k,0}}, \quad (8)$$

at $\xi = 0$.

In order to obtain the dependence of the ion velocity on time, it is need to solve Eq. (4).

From Eq. (4), the following differential equations of two variables are given:

$$\begin{cases} \frac{dp_k}{dt} = \frac{2\alpha V_k (1 + p_k^2)^{3/2}}{t \left[3\alpha p_k V_k \sqrt{1 + p_k^2} - (2\alpha + 1)t \right]}, \\ \frac{dV_k}{dt} = \frac{-2\alpha V_k}{3\alpha p_k V_k \sqrt{1 + p_k^2} - (2\alpha + 1)t}. \end{cases} \quad (9)$$

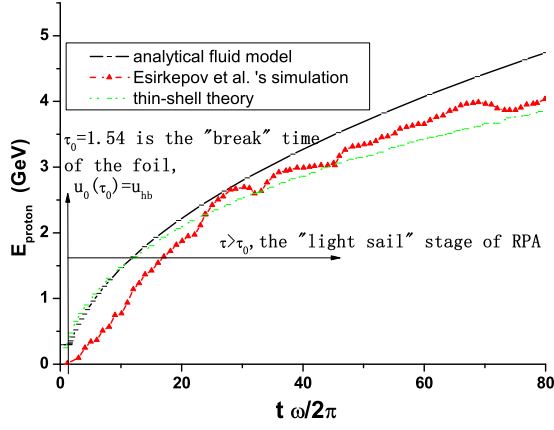


FIG. 1. (Color online) Comparison of our analytical fluid model with thin-shell model and Esirkepov et al.'s simulations for $\sigma/a \approx 0.1$, $a = 316$, $d = \lambda = 1\mu m$, $n_0 = 49n_c = 5.5 \times 10^{22}/cm^3$, and $\alpha = 1.8$ [9].

where $p_k = u_k \gamma_k$ is the normalized ion momentum and V_k satisfies:

$$V_k = \int_0^t \frac{p_k}{\sqrt{1+p_k^2}} dt - t \frac{p_k}{\sqrt{1+p_k^2}}. \quad (10)$$

Using matlab function *ode113* to solve Eq. (9) with the initial time τ_0 and initial velocity, $u_{k,0}$, the dependence of the ion energy on time has been calculated [14]. As an example, Figure 1 shows the comparison of our analytical fluid model with the thin-shell model and PIC simulations[3]. The results of our model consist well with that of the PIC simulations. When τ is larger than 40 times of the laser cycle, our results are a little larger than that of the PIC simulations. One of the reason is the loss of laser energy and the decreasing of the electron temperature for large τ in the simulations, while the electron temperature is assumed to be a constant in our model.

Combing Eqs. (1) and (5), it is obtained:

$$\hat{n}_k = \frac{1}{\Sigma Q_k} \frac{\left(\chi - \chi_0 - 2\alpha u_{k,0} \gamma_{k,0}^{3/2}\right)^{2\alpha}}{(4\alpha\beta_k)^\alpha}, \quad (11)$$

With Eqs. (3), (4) and (5), the electron density in the ion region is written as:

$$\hat{n}_e = (1 + \phi_1)^\alpha + \frac{1}{\rho\tau^2} \frac{\partial^2 \phi_1}{\partial \xi^2}, \quad (12)$$

Equation (12) shows the plasma can not be quasi-neutral at a finite time. However, the plasma tends to neutral as the time tends to infinite. It is obtained:

$$\lim_{\tau \rightarrow +\infty} n_e = \Sigma Q_k n_k, \quad (13)$$

With Eq. (11) and $\hat{n}_k(\xi = \xi_{i,f}) = 0$, the possible maximum ion energy at the ion front satisfies:

$$\int_0^{p_{k,m}} \gamma_k^{-\frac{3}{2}} dp_k = \int_0^{p_{k,0}} \gamma_k^{-\frac{3}{2}} dp_k + 2\alpha p_{k,0} \gamma_{k,0}^{\frac{1}{2}}, \quad (14)$$

where $\gamma_k^2 = 1 + p_k^2$, $p_{k,m} = u_{k,m}/\sqrt{1 - u_{k,m}^2}$ is the normalized limiting momentum of ions at the ion front: $\xi = \xi_{i,f}$ and $p_{k,0} = u_{k,0}/\sqrt{1 - u_{k,0}^2}$.

Different from the nonrelativistic case, it contains two acceleration modes and depends on the initial conditions: $u_{k,0}$ and α in the relativistic case. The critical condition is decided by:

$$\int_0^{p_{k,0}} \gamma_k^{-\frac{3}{2}} dp_k + 2\alpha p_{k,0} \gamma_{k,0}^{\frac{1}{2}} = \int_0^{+\infty} (1 + x^2)^{-3/4} dx \approx 2.622. \quad (15)$$

First, $\int_0^{p_{k,0}} \gamma_k^{-\frac{3}{2}} dp_k + 2\alpha p_{k,0} \gamma_{k,0}^{\frac{1}{2}} \lesssim 2.622$ for $p_{k,0} \lesssim 0.5064$ and $\alpha = 2$, which requires $a \lesssim 203$ for $n_0 = 49n_c$, $d = \lambda = 1\mu m$. Figure 2 shows the dependence of the plasma density, electric field and potential on ξ for $n_0 = 10^{22}/\text{cm}^3$, $\alpha = 2$ and $a = 70$. In this case, it is divided into two regions: the phase-stable acceleration region for $0 \leq \xi \leq \xi_{i,f}$ and the phase-stable deceleration region $\xi_{i,f} \leq \xi \leq 1$.

(I) In the phase-stable acceleration region, $0 \leq \xi \leq \xi_{i,f}$, the electric field $E \geq 0$ and the electron density is larger than the ion density. The maximum ion momentum $p_{k,m1}$ is limited and given Eq. (14) at the ion front $\xi_{i,f}$ ascertained by Eq. (4) with $u_k = u_{k,m1}$, where $u_{k,m1} = p_{k,m1}/\sqrt{1 + p_{k,m1}^2}$. At the ion front, the ion density is zero. The difference of the electron density and ion density decreases with time and tends to zero.

The potential shown by Figure 2 (b) in $0 \leq \xi \leq \xi_{i,f}$ gives an intuitionistic explanation of PSA. The ions coast down the slope of the potential, and the gradient, i.e., the electric field, becomes gently as the ions come to the bottom of the potential although few can reach there. Therefore the ions at higher potential will obtain more acceleration and the energy spread is improved. That is PSA. Different from the real gliding process, the ions can not pass through the bottom and climb up since the ion front is the limiting point and the ion density tends to zero at the bottom.

(II) In the phase-stable deceleration region, $\xi_{i,f} \leq \xi \leq 1$, the electric field $E \leq 0$ and the electron density is larger than the ion density too as shown in Figure 2. In this region, the ion momentum p_k satisfies: $p_{k,m1} \leq p_k \leq p_{k,m2}$, where $p_{k,m2} = p_k(\xi = 1)$ and given by Eq. (4) with $\xi = 1$. All the ions in this region are decelerated to $\xi = \xi_{i,f}$. The absolute value of the electric field decreases with the decreasing ξ and is zero at the ion front. Therefore the deceleration is also phase-stable.

With Figure 2 (b), for ions, the potential in $\xi_{i,f} \leq \xi \leq 1$ is a mountain with height of about 35GeV which is far larger than the maximum ion energy of about $\sqrt{p_{m,2}^2 + 1} - 1 \approx 7\text{GeV}$ at $\xi = 1$. Therefore, the ions will be decelerated at the potential uphill. Since the gradient, i.e., the value of electric field increases with the potential height, the deceleration is also phase-stable. As point out above, the limiting point is still the ion front $\xi = \xi_{i,f}$, and the ions in PSD region can also not pass through the bottom into PSA region. The ions in PSA and PSD region can not exchange from each other because of the zero-density dividing point $\xi = \xi_{i,f}$.

In this case, the maximum ion momentum is $p_{k,m1}$ at the ion front $\xi_{i,f} \lesssim 1$. Therefore, it is called phase-stable limited relativistic acceleration (PS-LRA). The ions in the two phase-stable regions can not exchange from each other and the ion front $\xi_{i,f}$ is the dividing line. Combining the condition of PS-LRA and the above discussion about PSA and PSD, the ions with an initial momentum of $p_{k,0}$ not large enough to get across the potential at $\xi = 1$ will drop in PSA region or PSD region and obtain a finite maximum energy ascertained by Eq. (14).

If the initial ion momentum satisfies:

$$\int_0^{p_{k,0}} \gamma_k^{-\frac{3}{2}} dp_k + 2\alpha p_{k,0} \gamma_{k,0}^{\frac{1}{2}} \geq 2.622. \quad (16)$$

the ions will not experience a potential well in PS-LRA and will coast down from the potential slope and drop into the bottomless abyss at $\xi = 1$ as shown by Figure 3 (d). $\xi = 1$ is called phase-lock-like position and the ions can obtain unlimited energy gain as shown in Figure 3 (b). It is called unlimited relativistic acceleration (URA), which is not phase-stable any more. Eq. (16) satisfies for $p_{k,0} \geq 0.5064$, i.e., $a \geq 203$ from Eq. (7) for $\alpha = 2$, $n_0 = 49n_e = 5.5 \times 10^{22}/\text{cm}^3$ and $d = \lambda = 1\mu\text{m}$. Figure 3 shows the dependence of the plasma density, electric field and potential on ξ for $a = 316$, $d = \lambda = 1\mu\text{m}$, $n_0 = 49n_e = 5.5 \times 10^{22}/\text{cm}^3$, and $\alpha = 2$. In this case, the electron density is smaller than the ion density and the acceleration

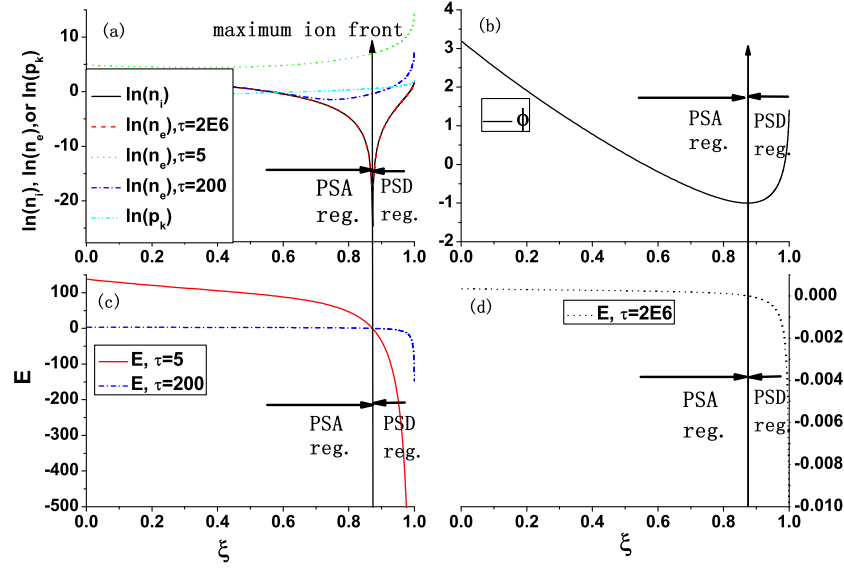


FIG. 2. (Color online) Phase-stable limited relativistic acceleration (PS-LRA) contains two regions: phase-stable acceleration region for $0 \leq \xi \leq \xi_{i,f} \approx 0.87$ and phase-stable deceleration region for $0.87 \leq \xi \leq 1$. (a) The density of ions and electrons for different time and the ion momentum VS the self-similar variable ξ . At the ion front $\xi_{i,f} = 0.87$, the ion density is zero. (b) The potential VS ξ . (c) and (d) The electric field \hat{E} VS ξ . $\hat{E} \geq 0$ for $0 \leq \xi \leq \xi_{i,f}$ and $\hat{E} \leq 0$ for $\xi_{i,f} \leq \xi \leq 1$. Here, $n_0 = 10^{22}/\text{cm}^3$, $\alpha = 2$ and $a = 70$.

is not phase-stable any more. In all the region, the electric field increases with ξ and is larger than zero. At a finite time, the electron front, where $n_e = 0$, $\xi_{e,f} \lesssim 1$, which is 0.971, 0.994, 0.99997 at $\tau = 5, 20, 2000$ separately. Therefore the possible maximum the ion momentum is shown by Figure 3 (a) and (b).

As discussed above, due to the initial ion momentum $p_{k,0}$ large enough, the ions can reach URA region. Therefore the ion momentum and the field have a sharp increase and tend to infinite, the ion density becomes a non-zero constant in $\xi \in (1 - \delta, 1]$ as $\tau \rightarrow \infty$, where δ is an infinitesimal. As shown in Figure 3 (b), at the phase-lock-like position and the limiting ion front $\xi = 1$, the ion can obtain unlimited energy gain and the ion density is non-zero. It is similar with the unlimited phase-lock ion acceleration as pointed out by Bulanov and coworkers[8] in the relativistic limit. In URA region, it is found that (I) the unlimited ion acceleration requires the initial ion momentum is large enough and should meet Equation

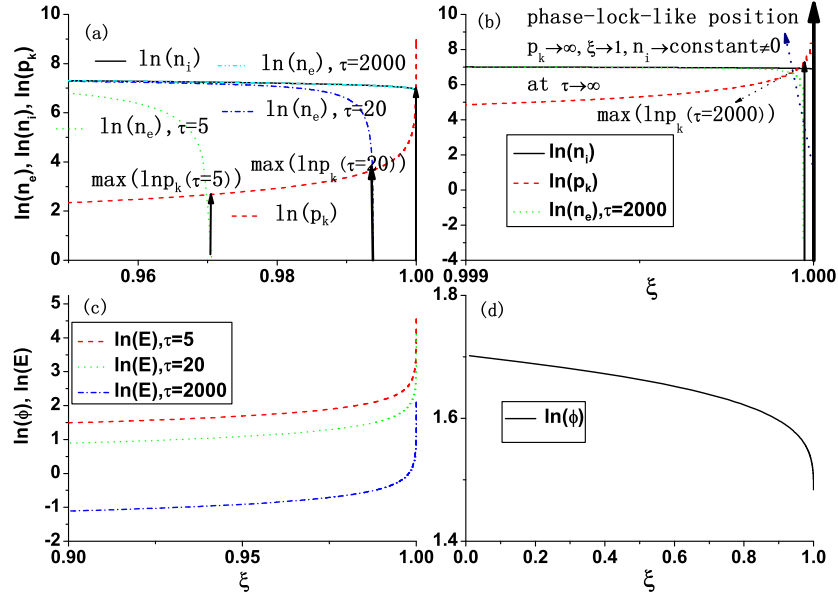


FIG. 3. (Color online) Unlimited relativistic acceleration (URA) with the phase-lock-like position $\xi = 1$. (a) The density of ions and electrons for different time and the ion momentum VS ξ . The ion density is non-zero at the limiting ion front: $\xi = 1$. The electron density is smaller than that of ions at any finite time. (b) The enlargement of (a) near $\xi = 1$. $\xi = 1$ is the phase-lock-like position where the ion momentum tends to infinity. (c) and (d) The electric field \hat{E} and the potential ϕ VS ξ . $\hat{E} \geq 0$ for all the region and the acceleration is not phase-stable any more. Here $a = 316$, $d = \lambda = 1\mu\text{m}$, $n_0 = 49n_c = 5.5 \times 10^{22}/\text{cm}^3$, and $\alpha = 2$.

(16); (II) it is not phase-stable any more; (III) the phase-lock-like position is $\xi = 1$, the limiting ion front; (IV) the ion density at the limiting ion front is non-zero.

In the conclusion, it has been given an analytical relativistic fluid model to describe the relativistic radiation pressure acceleration with the initial parameters from the hole-boring stage. The dependence of the ion velocity on the acceleration time can be obtained and is consistent with that of thin-shell model and PIC simulations. There are two acceleration modes: PS-LRA and URA with a critical initial ion momentum ascertained by an explicit formulation. In PS-LRA, the ions are trapped in a deep potential well and the maximum ion energy is limited and the ion front is the well bottom and $\xi_{i,f} \lesssim 1$. URA is not phase-stable any more and there is a phase-lock-like position in it. At the phase-lock-like position,

corresponding to the relativistic limit, the ions can obtain unlimited energy gain and the ion density is non-zero as time tends infinite. Although the unlimited ion acceleration can not be reached at any finite time, the ions can be accelerated to any large energy if the laser pulse is long enough. As an important result, you cannot obtain both PSA and URA. Therefore, if the laser parameters are large enough to obtain URA, the energy spread must be lost. If one wants to improve the energy spread with PSA, the maximum ion energy is limited.

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